Contents lists available at ScienceDirect
Journal of Financial Stability
journal homepage: www.elsevier.com/locate/jfstabil

Tail dependence and indicators of systemic risk for large US depositories☆

Eliana Balla[a,1], Ibrahim Ergen[a,1], Marco Migueis[b,*]

[a] Federal Reserve Bank of Richmond, United States
[b] Federal Reserve Board, United States

ARTICLE INFO

Article history:
Received 13 August 2013
Received in revised form 28 February 2014
Accepted 15 October 2014
Available online 27 October 2014

JEL classification:
G01
G18
G21

Keywords:
Extreme value dependence
Systemic risk
Systemically important financial institutions

ABSTRACT

In this study, we investigate the extreme loss tail dependence between stock returns of large US depository institutions. We find that stock returns exhibit strong loss dependence even in their limiting joint extremes. Motivated by this result, we derive extremal dependence-based systemic risk indicators. The proposed systemic risk indicators reflect downturns in the US financial industry very well. We also develop a set of firm-level average extremal dependence measures. We show that these firm-level measures could have been used to identify the firms that were more vulnerable to the 2007–2008 financial crisis. Additionally, we explore the performance of selected systemic risk indicators in predicting the crisis performance of large US depository institutions and find that the average stock return correlations are also good predictors of crisis period returns. Finally, we identify factors predictive of extremal dependence for the US depository institutions in a panel regression setting. Strength of extremal dependence increases with asset size and similarity of financial fundamentals. On the other hand, strength of extremal dependence decreases with capitalization, liquidity, funding stability and asset quality. We believe the proposed indicators have the potential to inform the prudential supervision of systemic risk.

1. Introduction

Financial crises over the last 20 years spread beyond the borders, industries or firms they originated in, causing widespread disruption to the broader economy. The Asian financial crisis of 1997 and the Russian default of 1999 spurred turmoil around the world, and the collapse of the hedge fund LTCM created similar concerns for the US economy. More recently, the global financial meltdown of 2007–2008 has reverberated across many countries and economic sectors, resulting in countless regulatory interventions. It has become clear that some institutions play a critical role in the financial system, due to their size, leverage, and interconnectedness with the rest of the financial industry. Therefore, measuring systemic risk in the financial system and identifying systemically interconnected financial institutions are critical tasks for policy makers and regulators.

In recent financial crises, extreme co-movement in asset prices and financial sector stock values was pervasive. As a result, correlations among different asset classes and, in particular, correlation among different banks experienced huge spikes, reaching unprecedented levels. Patro et al. (2013) explored the potential of stock return correlations of financial institutions as systemic risk indicators. They found that these correlations capture the downturns in the US financial system well. Building upon their analysis, we show that average stock return correlations can also be used to identify the firms that are more vulnerable in a financial crisis.

However, the Pearson correlation cannot be used in isolation to understand the dependence of stock returns. If stock returns followed a multivariate normal distribution, the Pearson correlation could summarize all information on their dependence. The literature has documented that stock returns exhibit skewness and significant excess kurtosis, which implies that they are fat

☆ The authors thank Jordan Nott and Laurel Mazur for excellent research assistance. The authors also thank Iftekhar Hasan, Jeffrey Gerlach, Jason Wu, Levent Guntay, two anonymous referees, and the participants of the FRB Richmond Applied Research Seminar and the FRB Philadelphia Risk Quantification Forum for valuable comments and discussion. *The opinions expressed in this manuscript belong to the authors and do not represent official positions of the Federal Reserve Bank of Richmond, the Federal Reserve Board or the Federal Reserve System.*

* Corresponding author at: Board of Governors of the Federal Reserve System, Division of Banking Supervision and Regulation, 20th and C Streets, NW, Mail Stop 1809, Washington, DC 20551-001, United States.

E-mail addresses: Eliana.Balla@rich.frb.org (E. Balla), Ibrahim.Ergen@rich.frb.org (I. Ergen), mmigueis@gmail.com (M. Migueis).

[1] Address: Federal Reserve Bank of Richmond, Baltimore Office, 502 S. Sharp Street, Baltimore, MD 21201, United States.

http://dx.doi.org/10.1016/j.jfs.2014.10.002

tailed,[2] and thus not normally distributed. Also, the Pearson correlation only measures linear dependence between assets' returns and, therefore, does not account for nonlinearities. Moreover, the Pearson correlation may be particularly limited in assessing dependence during financial crises. It cannot fully capture an increase in dependence in the joint tails of distributions, because it measures "average" dependence, with no particular focus on the extremes of distributions and, therefore, is dominated by the observations around the mean.[3] We believe that understanding loss dependence in the joint extremes of the loss distributions is crucial in understanding systemic risk because the prevalence of dependence in the extreme tails of loss distributions is indicative of high contagion potential between financial institutions. Also, it is essential to measure co-movement of financial institution returns when institutions experience stress, as it can be very different from the co-movement during normal times. In particular, co-movement when institutions experience stress may become stronger. Thus, we believe that extremal dependence measures can be very valuable tools in systemic risk measurement. Patro et al. (2013) also acknowledges the importance of investigating tail dependencies to measure systemic risk, but left it as an area for future research. In this study, we fill this gap and propose systemic risk measures derived from multivariate extreme value theory (EVT), which can capture the tail dependencies between stock returns of large U.S. depository institutions. We believe that these measures can be used as complementary tools in monitoring systemic risk and we demonstrate their additional value to such analysis.

We construct indicators of systemic risk based on extremal dependence measures formally developed by Ledford and Tawn (1996, 1997, 1998). The extremal dependence measure χ is defined as:

$$\chi = \lim_{q \to 1} Pr(L_1 > L_{1,q} | L_2 > L_{2,q})$$

where L_1, L_2 stand for two loss variables and $L_{1,q}, L_{2,q}$ stand for their respective marginal qth quantiles. If $\chi = 0$, L_1 and L_2 are said to be asymptotically independent. If $\chi > 0$, L_1 and L_2 are said to be asymptotically dependent and χ measures the strength of the asymptotic dependence. For example, Sibuya (1960) showed that marginals of multivariate normal distribution are asymptotically independent as long as the correlation coefficient is less than unity.[4] Poon et al. (2004) developed a simple estimation methodology to identify the existence of asymptotic dependence, as well as to measure its strength when it exists. We will rely on this method for our empirical analysis.

We propose two complementary systemic risk indicators. The first indicator is the proportion of asymptotically dependent depository institution pairs to the total number of depository institution pairs in our sample, thus measuring the prevalence of asymptotic dependence between large US depository institutions. Chan-Lau et al. (2004) investigated the strength of financial contagion in international stock markets by calculating the proportion of asymptotically dependent country pairs during the Mexican peso crisis and the Asian crisis. We borrow this measure from the international financial contagion literature to measure the systemic risk in the US banking system. The other measure we propose is the average strength of asymptotic dependence, average χ, across all pairs of depository institutions. This latter measure can provide insights regarding the strength of extremal dependence in the US banking system, beyond the proportion of asymptotically dependent institutions. Our results show that these indicators track reasonably well periods of financial market turmoil and periods of market stability.

Besides accurately monitoring overall systemic risk, another challenge for regulators attempting to minimize systemic risk is identifying systemically important financial institutions (SIFIs). Appropriately measuring the vulnerability and contagion potential of firms in a systemic crisis would allow regulators to better target policies to contain systemic risk. In the aftermath of the 2007–2008 financial crisis, US and international policy makers are enacting new regulations for financial institutions perceived as "too big to fail", due to their size, interconnectedness, complexity, lack of substitutability, or global scope.[5]

Because systemic risk is a complex phenomenon, it is not surprising that many indicators of systemic importance have been proposed in the literature. Prominent among these measures are the CoVaR (Adrian and Brunnermeier, 2011), the Marginal Expected Shortfall ("MES", Acharya et al., 2010), Distressed Insurance Premium ("DIP", Huang et al., 2009) and the granger causality based measures of Billio et al. (2012). We believe that the co-existence of multiple approaches to measuring systemic importance and institution-specific vulnerability to crisis could enhance the prudential supervisory toolkit and the regulation of systemic risk. We add to the literature by developing two complementary firm-level measures of average tail dependence, based on the stock price co-movements under conditions of joint stress. Our first measure is the proportion of other institutions that are asymptotically dependent with an institution. Our second measure is the average χ of an institution (i.e. the average χ of the bank pairs including the institution of interest). These measures of tail co-movement can help understand the vulnerability and the contagion potential of a financial institution during a financial crisis.

Then, we formally explore whether our measures of tail co-movement could have predicted the vulnerability of a financial institution to the 2007–2008 financial crisis. We calculated these measures before the beginning of the crisis and investigated their predictive power over crisis period stock returns. We find that these measures are statistically significant predictors of crisis period stock returns. In order to demonstrate the value added by the measures proposed in this study, we also tested the predictive ability of Patro et al. (2013) correlation measures, Acharya et al. (2010) MES measure, and the CAPM beta. We find that our measures predict crisis period returns better than the CAPM beta and the MES. We also find that correlation based measures are good predictors of vulnerability to a financial crisis. This finding provides more evidence for the usefulness of the measures proposed by Patro et al. (2013). Nevertheless, we demonstrate that our measures predict financial crisis returns better than the correlation based measures when the crisis period is chosen more narrowly, to coincide with the largest financial crisis losses. In particular, when crisis performance is measured with peak-to-trough returns instead of cumulative returns,

[2] See Mandelbrot (1963) and Fama (1965).

[3] The Pearson correlation coefficient equally weights all observations in a sample. Since observations tend to be clustered close to the mean and scarce in the tails, observations close to the mean tend to dominate the calculation.

[4] Sibuya (1960) originally suggested asymptotic dependence and independence as abstract theoretical concepts. It was Ledford and Tawn (1996, 1997, 1998) who developed the theoretical characterization of joint tails for multivariate extremes which enabled estimation of the tail dependence structure through empirical diagnostic tests.

[5] Both the Financial Stability Oversight Council (FSOC) (a creation of the Dodd–Frank Act in the United States) and the Basel Committee weighed in on the criteria for the designation of systemically important institutions in Q4 2011. For the United States, the Dodd–Frank Act (Section 113) lists statutory considerations for the designation of systemically important institutions. For additional details on global designation, see Basel Committee (2011). We include many of the FSOC and Basel Committee proposed designation criteria in Section 4 when we look at the firm-specific financial indicators that can predict systemic interconnectedness for financial institutions.

our measures perform significantly better. These results indicate that the firm-level measures proposed in this study may predict the vulnerability of financial institutions to extreme market crashes better than the correlation measures. Therefore, we suggest that risk managers use them in monitoring the riskiness of firms as complements to correlation analysis.

Zhou (2010) and Moore and Zhou (2012) suggest a methodology to identify "systemically important" institutions similar to the firm-level average tail dependence measures we propose in this paper. We believe our approach to measure tail dependence is preferable to theirs since our methodology explicitly tests for the existence of asymptotic dependence following Poon et al. (2004). In Moore and Zhou (2012) asymptotic dependence is assumed to exist if their estimate of χ is above 0.15 and to not exist otherwise, independently of the precision of the estimate.

Another important contribution of this paper to the tail dependence literature is that we take a step beyond measuring extremal dependence, and try to identify factors that are predictive of tail dependence. Identifying such factors can inform the selection of institutions that are systemically important. In this analysis, we consider an array of institutional characteristics traditionally used in the bank performance literature. We find that strength of extremal dependence increases with asset size and similarity of financial fundamentals. On the other hand, strength of extremal dependence decreases with capitalization, liquidity, funding stability and asset quality.

Finally, we also calculated the proposed systemic risk indicators using the filtered returns from a three factor Fama-French model. We observe that the increase in systemic risk around 1998, which coincides with the Asian crisis, Russian default and the collapse of hedge fund LTCM, was almost entirely driven by systematic market factors. On the other hand, the recent financial crisis, which began as a crisis of subprime mortgages, was in large part driven by other factors not captured by the Fama-French model. We think that these observations are intuitive as the crisis in 1998 did not originate in the US banking industry, whereas the recent financial crisis did. Also, we find that after the second half of 2009 the factors driving the tail dependence among banks appear to become systematic in nature.

The remainder of the paper is organized as follows. Section 2 introduces our measures of systemic risk. In Section 3, we demonstrate that our measures could have been used to identify the firms that were more vulnerable to the 2007–2008 financial crisis. In Section 4, we analyze how balance sheet variables can be used to predict the strength of asymptotic dependence between stock returns of bank pairs. In Section 5, we examine the tail dependence between filtered returns and discuss how systematic and non-systematic factors drove tail dependence in recent episodes of crises. Section 6 concludes.

2. Extremal dependence and systemic risk measures

Multivariate extreme value theory (EVT) is one of the promising tools in the literature to measure the dependence of extremes and address the drawbacks of linear correlation analysis. Numerous studies have applied multivariate EVT techniques to the analysis of the tail dependence in financial markets (e.g. Chan-Lau et al., 2004; Hartmann et al., 2004; Longin and Solnik, 2001; Poon et al., 2004; Starica, 1999). Unlike earlier studies, Poon et al. (2003, 2004) made clear the distinction between asymptotic dependence and asymptotic independence, while analyzing the tail dependence structure of five major stock indices. Such distinction is very important, as it indicates the existence or lack of dependence in the limiting joint extremes of the loss distributions. Such extremes may have never been observed in the historical data, but dependence at those levels is what matters most when managing risk against catastrophic events. For example, Ergen (2014) empirically demonstrates that the diversification benefits, measured as the reduction in extreme tail risk, are on average three times larger for asymptotically independent emerging market pairs than for asymptotically dependent emerging market pairs.

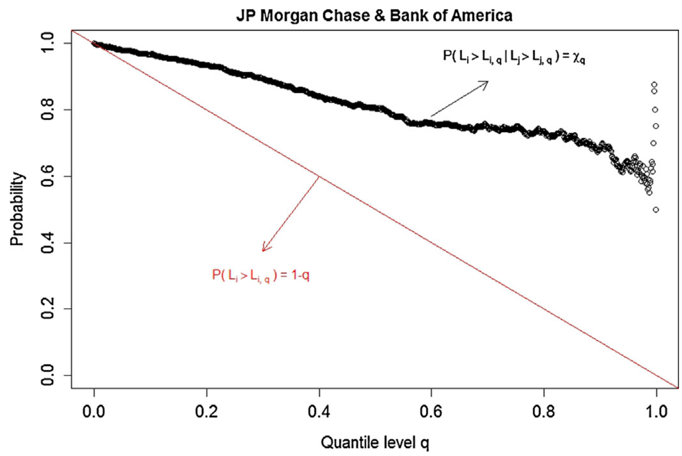

Fig. 1. An example of an asymptotically dependent institution pair.

2.1. A graphical introduction to asymptotic dependence

Let L_1 and L_2 be two loss random variables. And let χ_q be the probability of one of the loss variables L_i being above the qth quantile of its marginal distribution, $L_{i,q}$, conditional on the other loss variable L_j being above its qth quantile, $L_{j,q}$.

$$\chi_q = Pr(L_1 > L_{1,q}|L_2 > L_{2,q}) = Pr(L_2 > L_{2,q}|L_1 > L_{1,q})$$

The tail dependence measure χ is defined as:

$$\chi = \lim_{q \to 1} \chi_q \tag{1}$$

If $\chi > 0$, the two loss variables, L_1 and L_2, are said to be asymptotically dependent, while if $\chi = 0$, they are said to be asymptotically independent. Asymptotic dependence between two variables means that, no matter how far into the tail of one variable we go, the conditional probability of the other variable simultaneously experiencing a tail event at least as severe (in quantile terms) never reaches zero. The limiting extreme of the loss distribution of a firm is likely to correspond to losses so large as to cause the firm to go bankrupt. Thus, the χ of a pair of firms can loosely be interpreted as the conditional probability of default of a firm, when the other firm defaults.[6]

As an example of an asymptotically dependent pair of institutions, in Fig. 1, we plot χ_q for the stock returns of JP Morgan Chase and Bank of America, as a function of the quantile level q.[7] The unconditional probability of a loss greater than the qth quantile is also plotted as the line $p = 1 - q$.

There is strong positive association between the losses of these two institutions, as the conditional probability of an institution experiencing a loss larger than $L_{i,q}$ is always much greater than the unconditional probability. Moreover, as the quantile level

[6] For example, Moore and Zhou (2012) and Zhou (2010) provide this interpretation of χ.

[7] The sample period used for the graphical analysis in Figs. 1 and 2 goes from the beginning of 2006 to the end of 2011.

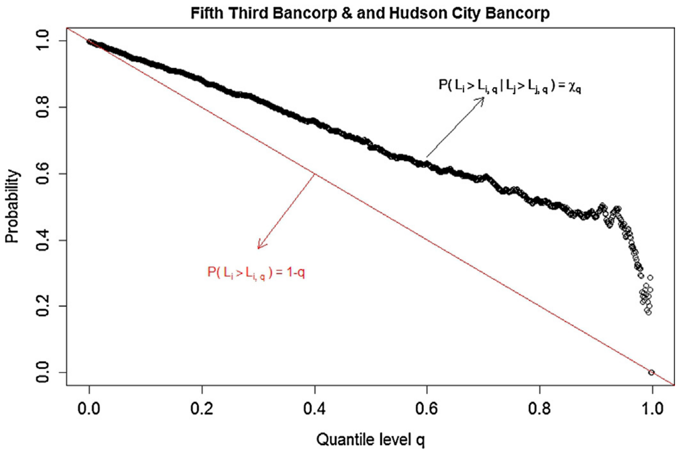

Fig. 2. An example of an asymptotically independent institution pair.

increases, this positive association of losses never dies off. Instead, as q approaches one, the conditional probability is converging to a positive value above 0.6. Thus, the stock returns of these two banks are asymptotically dependent.

As an example of an asymptotically independent pair of institutions, in Fig. 2 we plot the χ_q for the stock returns of Fifth Third Bancorp (FITB) and Hudson City Bancorp (HCBK). Fig. 2 illustrates how asymptotic independence is different from asymptotic dependence, represented in Fig. 1, while also showing that asymptotic independence is not the same thing as exact independence. Again, there is positive association between these two institutions, as χ_q is always higher than the unconditional probability (and thus the stock returns of these two banks are not totally independent). However, for this pair, as q gets closer to one, the conditional probability approaches the unconditional probability and, therefore, zero. Therefore, the stock returns of these banks are asymptotically independent.

Figs. 1 and 2 are presented in order to introduce the concept of asymptotic dependence in a graphical way. These plots can serve as diagnostic tools, but cannot be used for testing asymptotic dependence in a formal way. Determining the extremal dependence structure for two variables requires rigorous statistical analysis and hypothesis testing. In this paper, we follow the methodology developed by Poon et al. (2004) to determine the extremal dependence structure of the stock returns of large US depository institutions.[8]

2.2. Formal statistical estimation of extremal dependence

Following Poon et al. (2004), we begin by transforming the marginal distributions of our loss variables to unit Frechet marginal distributions. Conversion of univariate marginal variables to a common distribution is standard in tail dependence studies because this conversion removes the influence of marginal distributions from dependence calculations.[9] In the literature, uniform distributions and Frechet distributions are widely used to standardize marginal distributions. In this study, we transform the loss variables to have unit Frechet marginals in order to follow the Poon et al. (2004) methodology, which has become the standard method in estimating the extremal dependence measure χ. Loss variables are converted to unit Frechet margins by the following probability integral transform:

$$X = \frac{-1}{\log F_{L_1}(L_1)} \quad \text{and} \quad X_2 = \frac{-1}{\log F_{L_2}(L_2)} \tag{2}$$

where F_{L_1} and F_{L_2} are the distribution functions for loss variable L_1, and L_2,[10] and $-1/\log(x)$ is the quantile function of the unit Frechet distribution. Since $-1/\log(\mathrm{F}(L))$ is a monotonically increasing function of L, this transformation has no impact on the order of the data and therefore has no impact on χ. This is because χ is a quantile-based measure and, thus, invariant to monotonically increasing transformations of the data.

For Frechet distributed variables, it is known that:

$$\lim_{r\to\infty} Pr(X \le r) = 1 - \frac{1}{r} \tag{3}$$

In other words, as r increases, the $(1-1/r)$th quantile of Frechet distribution converges to r. Also, Ledford and Tawn (1996) show that the joint tail region of two unit Frechet distributed variables satisfies:

$$Pr(X_1 > r, X_2 > r) = l(r) r^{-1/\eta} \tag{4}$$

where $l(r)$ is a slowly varying function,[11] and $\eta \in (0,1]$ is the coefficient of tail dependence. Using these two properties, we can derive the χ for two unit Frechet distributed variables X_1 and X_2. First, we replace q in the definition of χ by $1 - 1/r$ and obtain:

$$\lim_{r\to\infty} l(a * r)/l(r) = 1 \; \forall \, a > 0$$

Then, using (3), we replace the $(1-1/r)$th quantile of the unit Frechet variables by r. Further, we note that $(1 - 1/r)$ approaches one, if and only if r approaches infinity. Therefore, we obtain:

$$\chi = \lim_{r\to\infty} Pr(X_1 > r | X_2 > r) = \lim_{r\to\infty} \frac{Pr(X_1 > r, X_2 > r)}{Pr(X_2 > r)}$$

Using (3) for the denominator and (4) for the numerator, we obtain:

$$\chi = \lim_{r\to\infty} \frac{l(r) \times r^{-1/\eta}}{1/r} = \lim_{r\to\infty} l(r) \times r^{1-1/\eta} \tag{5}$$

Therefore, $\eta = 1$ corresponds to asymptotic dependence and $\eta < 1$ corresponds to asymptotic independence. Also, when $\eta = 1$ and we have asymptotic dependence, χ is equal to the limit of the slowly varying function $\mathrm{l}(r)$. Hence, the problem of estimating the asymptotic dependence structure reduces to estimating the coefficient of tail dependence η and the limit of the slowly varying function.

In order to accomplish this estimation, Poon et al. (2004) note that the expression on the right hand side of (4) is the same as a power law specification for univariate heavy tailed variables.[12] As

[8] The formal results from the Poon et al. (2004) methodology for the JP Morgan and the Bank of America pair and the Fifth Third and Hudson City pair are, respectively, that the null hypothesis of asymptotic dependence cannot be rejected for JPMC and BAC, and χ is estimated to be 0.61; and that asymptotic dependence is rejected for Fifth Third and Hudson City and, therefore, they are asymptotically independent ($\chi = 0$).

[9] According to Sklar's theorem any multivariate distribution function can be separated into its marginal distributions and another function, known as the copula, which reflects the dependence between the variables. Since the tail dependence estimation method used in this paper is independent of any parametric copula assumption, a detailed discussion of copula methods is not required. For an introduction to copula modeling please see Nelsen (2006) and Trivedi and Zimmer (2007). Also, for a complete discussion of the importance of separating the marginal distributions and the dependence structure see Embrechts et al. (1999).

[10] The empirical cdf is used as the distribution function for losses.

[11] A slowly varying function is defined by $\lim_{r\to\infty} l(a^*r)/l(r) = 1 \; \forall \, a > 0$.

[12] In the EVT literature and for the purposes of this study, the definition of heavy-tailed variable relies on the decay rate of the tail for the cumulative distribution function. If the tail of a distribution decays slower than the exponential rate, the distribution can be categorized as a heavy-tailed distribution. Therefore, they are commonly referred to as sub-exponential distributions.

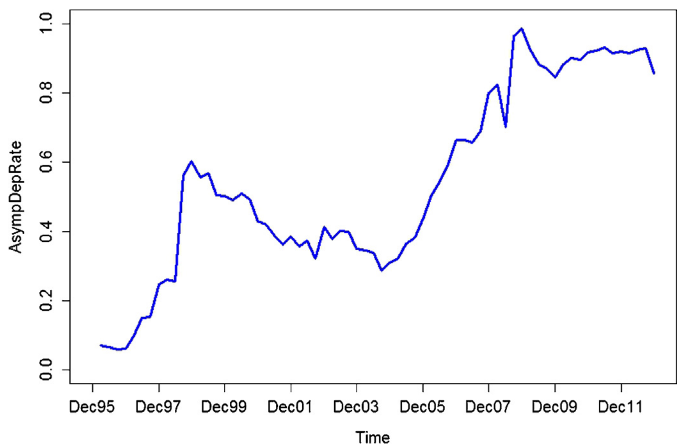

Fig. 3. *AsympDepRate* over time using six year rolling window samples.

discussed in Appendix A, heavy-tailed distributions follow a power law beyond a high threshold, which can be written as:

$$\overline{F_T}(t) = Pr(T > t) \cong l(t)t^{-1/\xi} \text{ for } t > u \quad (6)$$

where $l(t)$ is a slowly varying function, u is a high threshold and ξ is the tail parameter. Therefore, Poon et al. (2004) proposed to construct a new variable, $T = \text{Min}(X_1, X_2)$, so that (4) can also be written as:

$$Pr(X_1 > r, X_2 > r) = Pr(T > r) = l(r)r^{-1/\eta} \quad (7)$$

Comparing this with (6), we notice that T is a heavy tailed univariate variable with tail parameter η. The tail parameter can be estimated by the Hill's estimator and inference can be performed using its asymptotic properties. Details on the estimation of the tail parameter can be found in Appendix A.

In order to test for asymptotic dependence, we test the null hypothesis $\eta = 1$. If the null hypothesis of asymptotic dependence ($\eta = 1$) cannot be rejected, then χ is calculated as the limit of the slowly varying function $l(r)$. Details of this calculation are also shown in Appendix A.

2.3. Systemic risk and firm-level extremal dependence measures

After the asymptotic dependence hypothesis is tested and χ is calculated for each pair of firms in our sample, we define our two systemic risk indicators and two firm-level extremal dependence measures. Let:

$$AsympDep_{ij,t} = \begin{cases} 1 & \text{if} \quad \chi_{ij,t} > 0 \\ 0 & \text{if} \quad \chi_{ij,t} = 0 \end{cases}$$

The first systemic risk indicator, *AsympDepRate*, is defined as the ratio between the number of asymptotically dependent institution pairs and the total number of institution pairs in our dataset, for a given time period.

$$AsympDepRate_t = \frac{\sum_i \sum_{j \neq i} AsympDep_{ij,t}}{N \times (N-1)}$$

This ratio measures the system-wide prevalence of dependence in the joint extremes of the loss distributions of bank pairs. Thus, *AsympDepRate* quantifies the system-wide potential for simultaneous extreme loss events in multiple depository institutions. In Fig. 3, we present a time series plot of *AsympDepRate*. To obtain the asymptotic dependence for time t, data from the six years preceding time t is used. When we repeat the calculation for time $t+1$, we simply roll the sample one quarter forward. For example, asymptotic dependence is estimated for each pair of institutions in the sample using data from January 1990 to December 1995, and the *AsympDepRate* calculated from this period is plotted at December 1995. Then, the procedure is repeated using data from April 1990 to March 1996 and the *AsympDepRate* is plotted at March 1996, and so on. We have chosen a six year sample window for each iteration because extreme value theory applications generally require large sample sizes in order to accurately measure tail dependence.[13] The methods only use loss events that lie in the tails of the data and, therefore, enough extreme loss events are necessary to obtain precise estimates of asymptotic dependence.

The *AsympDepRate* systemic risk indicator captures financial crises well. At the end of 1995 the indicator was below 10% but started climbing, reaching nearly 25% after the Asian financial crisis. Afterwards, there is a more significant increase around the collapse of the hedge fund Long Term Capital Management (LTCM) on September 1998 and a slow decline, up until 2004. Finally, the indicator gradually increased during the 2005–2008 period, peaking at an unprecedented 95% during the heights of the financial crisis.

The measurement of χ provides more information beyond what is learned by establishing that two variables are asymptotically dependent. When two loss distributions are asymptotically dependent, we know that the conditional probably of joint tail losses never goes to zero, no matter how far into the joint tail we go. χ measures the size of this conditional probability. When χ is high, we not only know that there is a positive probability that extreme tail losses can occur simultaneously, but also that this probability is high. We believe that an aggregate measure of χ can be a useful systemic risk indicator, complementing *AsympDepRate*.[14] The second systemic risk measure we propose, *AvgChi*, is defined as the average of χ over all possible institution pairs in our dataset for the given time period.

$$AvgChi_t = \frac{\sum_i \sum_{j \neq i} \chi_{ij,t}}{N \times (N-1)}$$

In Fig. 4, we plot *AvgChi*, calculated using a six year rolling window.

AvgChi and *AsympDepRate* are fairly synchronized. The main difference between the two is their scale. *AsympDepRate* is calculated from a binary variable that indicates existence of asymptotic dependence. *AvgChi* measures the strength of that dependence, if it exists. Analysis of the *AvgChi* indicator illustrates that the latest financial crisis was substantially more severe than the previous crises in the end of the 1990s.

The ability of the systemic risk indicators we developed to predict financial crises with substantial lead time may appear limited. Unfortunately, this is a common feature shared by other indicators of systemic risk that are based on market return information. That being said, the warning potential of our indicators should be evaluated relative to the information available ex ante. If we were in December 2006, we would not know what will happen to these measures in 2007 and forward. Instead, we would only be able to see the historical values up until December 2006 and, from this perspective, the values of these indicators in December 2006 are at their historically highest levels. The fact that the indicators reach

[13] With a sample of 6 years, we have approximately 1500 daily returns, which in turn result in approximately 75 extreme tail observations to be used in Hill's estimation for testing the asymptotic dependence hypothesis.

[14] DiTraglia and Gerlach (2013) corroborate the importance of χ. They showed that χ is priced by investors during good times (i.e. investors require a premium to invest in firms with large χ).

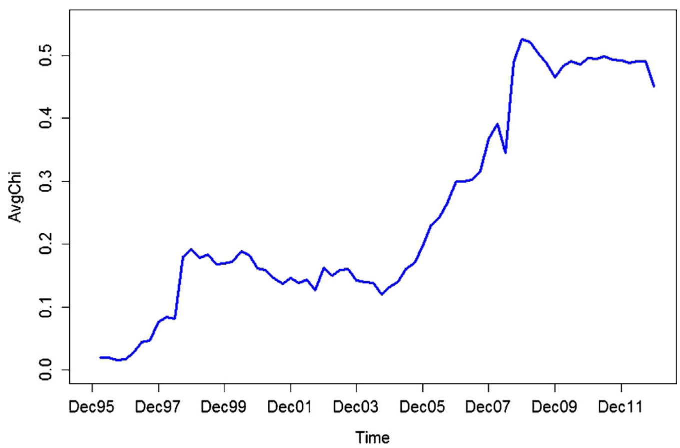

Fig. 4. *AvgChi* over time using six year rolling window samples.

even higher levels in 2008, during the crisis, does not imply that the indicators were not signaling high levels of tail dependence during 2006 and 2007. We still do not interpret our indicators as predictive of future crisis; based on the graphical analysis of the indicators, that would be a strong statement. However, it is obvious that these indicators track periods of financial market turmoil and periods of market stability reasonably well.

In order to measure the firm-level extremal dependence of individual institutions, we develop measures that are similar to the systemic risk indicators. A limitation of using extremal dependence measures to evaluate an institution's systemic importance is that these measures do not allow us to ascertain the direction of asymptotic dependence. Despite this limitation, we believe that it is useful to determine which institutions exhibit asymptotic dependence to a larger set of other financial institutions and which institutions exhibit, on average, stronger asymptotic dependence, as these institutions may be more vulnerable to market swings and more likely to propagate tail losses. Another limitation of our firm-level measures is that they do not take into account the size and leverage of the firms, and so they cannot be directly translated to an estimate of capital shortfall under stressful conditions.

The first measure of firm-level extremal dependence reflects the percentage of institutions that are asymptotically dependent with a given institution.

$$AsympDepRate_{i,t} = \frac{\sum_{j \neq i} AsympDep_{ij,t}}{(N-1)}$$

The second firm-level extremal dependence measure we propose results from averaging χ for all institution pairs an institution is involved in.

$$AvgChi_{i,t} = \frac{\sum_{j \neq i} \chi_{ij,t}}{(N-1)}$$

The usefulness of these two measures as indicators of the vulnerability of large US depository institutions to a financial crisis is studied in detail in the next section.

3. Predictive power over financial crisis returns

We follow the literature in selecting institutions with market capitalization greater than \$5 billion at the end of June 2007 (Acharya et al. (2010); Brownless and Engle (2012)). Among the diverse set of large financial institutions with capitalization above \$5 billion, our sample consists of the 29 institutions that are classified as depositories based on their SIC codes. We focused on depository institutions for several reasons. First, analysis of a heterogeneous sample of firms from multiple industries may lead to inconclusive results regarding the significance of a given measure of vulnerability to a crisis, while focusing on a particular industry may lead to a clearer picture. By focusing on depositories only, we were able to achieve up to 75% adjusted R^2 in our regressions. Second, depositories are the firms that have explicit deposit insurance and, thus, are subject to a moral hazard problem. Finally, one of the contributions of our study is an analysis of factors predictive of tail dependence. Such novel analysis can only be easily performed for depository institutions, which have been subject to regulatory reporting for the whole period of our sample. In Appendix B, Table B1 presents the variables and their definitions used in the empirical analysis presented in Sections 3 and 4.

We obtain daily returns from CRSP as well as market capitalization and quarterly financial data from Compustat. The panel of firms is unbalanced, as not all companies have continuously been trading during the sample period of January 2, 1990 to March 30, 2012. Some companies such as Washington Mutual failed or were acquired during the sample period. Further information on the dataset construction and an account of the institutions that do not have returns for the entire sample period is included in Appendix B Table B2.

Descriptive statistics for the daily percentage returns of the firms are presented in Table 1. Not surprisingly, the minimum daily returns are very extreme for those firms that failed during the crisis (National City Corp with −63.34%, Sovereign bank with −72.16%, Wachovia Corp with −81.6%, Washington Mutual with −90.51%). The return of most stocks presents positive skewness and significant excess kurtosis. This skewness and excess kurtosis indicates stocks returns have "fat tails" and, thus, are not normally distributed. Jarque Bera tests are also presented in Table 1 to provide formal evidence that the stock returns in our sample are not normally distributed.

Next, we explore the predictive power of our proposed firm level average tail dependence measures over the stock return of firms during the recent financial crisis. If these measures have explanatory power over the performance of financial institutions during financial crises, they can potentially be used by regulators to identify the firms that are more vulnerable to systemic crisis.

In testing the out-of-sample predictive power of our measures, we present a case study where we follow the empirical approach taken by Acharya et al. (2010) of calculating systemic vulnerability indicators for a pre-crisis period, and then evaluating their ability to predict out-of-sample stock returns during the crisis period. We consider two potential periods for the financial crisis. First, following Acharya et al. (2010) and Billio et al. (2012), we consider the financial crisis to span from July 2007 to December 2008. Thus, we calculate our measures as of June 30, 2007, and then use the measures to predict cumulative stock returns from July 2007 to December 2008.

Second, we repeat the analysis defining the crisis period as January 2008 to December 2008. In this case, we calculate our measures as of December 31, 2007. The latter definition of crisis corresponds to the period where most of the extreme losses, which can be referred to as tail events, occurred. This is evidenced by the descriptive statistics presented in Table 2. In Panel A, we present firm-level vulnerability indicators as of June 2007 and return variables (cumulative and peak-to-trough returns) for the crisis period from July 2007 to December 2008. In Panel B, we present firm-level vulnerability indicators as of December 2007 and the return variables are based on the crisis period from January 2008 to December 2008. As expected, all of the firm-level vulnerability indicators

Table 1
Descriptive statistics of daily percentage returns.
Notes: The raw return data is the daily percentage return calculated as $R_t = (P_t - P_{t-1})/P_{t-1}$, where P_t is the market price of a stock on the closing of day t. For some firms the data is not complete as can be seen from the first column. For details on institutions that do not have returns for the entire sample period, please see Appendix B. The high p-values in all the Jarque–Bera tests reflect that normality of returns was rejected for all institutions.

	N	Minimum return (%)	Maximum return (%)	Mean	Median	Standard deviation	Skewness	Excess kurtosis	JB test (p-value)
Bank of America	5796	−28.97	35.27	0.04	0	2.85	0.82	25.31	155,457.09
BB&T	5796	−23.36	23.61	0.05	0	2.17	0.66	15.19	56,226.92
Bank of NY Mellon	5796	−27.16	24.81	0.06	0	2.47	0.56	12.74	39,559.66
Citigroup	5796	−39.02	57.82	0.06	0	3.08	1.33	42.7	442,269.18
Commerce Bancorp	4597	−12.12	26.92	0.09	0	2.32	0.5	7	9582.38
Comerica	5796	−20.3	20.69	0.04	0	2.27	0.3	12.31	36,742.58
Huntington Bancshares	5796	−30.59	50.07	0.05	0	3.19	2.08	40.46	399,912.16
Hudson City Bancorp	3389	−14.01	15.68	0.06	0	1.89	0.18	9.74	13,432.66
JPMorgan Chase	5796	−20.73	25.1	0.06	0	2.6	0.7	10.73	28,294.33
Keycorp	5796	−50.59	54.25	0.03	0	2.78	0.16	61.75	921,450.38
Marshall & Ilsley	5421	−26.03	39.01	0.05	0	2.74	1.2	33.13	249,480.67
M&T Bank	5796	−15.61	21.06	0.06	0	1.8	0.67	14.57	51,775.76
National City Corp	4790	−63.34	65.14	0.01	0	2.77	−0.05	154.09	4,742,780.98
Northern Trust	5796	−18.81	30.91	0.06	0	2.15	0.85	15.36	57,712.74
New York Comm. Bank	4810	−13.8	13.95	0.07	0	2.03	0.11	6.56	8651.62
People's United	5796	−19.05	26.67	0.07	0	2.43	0.78	11.77	34,094.28
PNC	5796	−41.4	37.09	0.05	0	2.36	0.52	36.97	330,660.97
Regions Financial	5796	−41.07	48.41	0.04	0	2.88	1.4	45.66	505,829.41
Synovus Financial	5796	−25.97	28.22	0.05	0	2.82	0.55	14.5	51,106.05
Sovereign Bank	4807	−72.16	69.53	0.07	0	3.19	−0.08	106.43	2,270,524.29
Suntrust Banks	5796	−27.17	30.56	0.05	0	2.53	0.53	21.79	114,993.68
State Street Corp	5796	−59.04	31.35	0.08	0	2.64	−1.04	58.14	818,068.05
Unionbancal Corp	4750	−30.5	15.19	0.07	0	2.05	−0.33	15.36	46,831.85
US Bancorp	5796	−18.17	29.39	0.07	0	2.23	0.75	17.04	70,750.83
Wachovia Corp	4790	−81.6	90.22	0.05	0	3.31	3.96	246.81	12,180,184.62
Wells Fargo & Co.	5796	−23.82	32.76	0.07	0	2.43	1.61	27.67	187,584.35
Washington Mutual	4724	−90.51	48.76	0.01	0	3.2	−3.99	165.37	5,399,843.72
Western Union	1572	−29.01	21.03	0.01	0	2.45	−0.84	19.64	25,515.18
Zions Bancorporation	5796	−24.54	27.56	0.07	0	2.76	0.65	17.95	78,308.51

Table 2
Descriptive statistics of variables used in regressions.
Notes: Panel A presents firm-level vulnerability indicators, total assets and leverage as of June 2007 and return variables (cumulative and peak-to-trough returns) for the crisis period from July 2007 to December 2008. Panel B presents firm-level vulnerability indicators, total assets and leverage as of December 2007 and the return variables are based on the crisis period from January 2008 to December 2008. Note that the statistics of cumulative return in Panel B are for a shorter time period. So, for example, the 18 month equivalent of the 38.8% average decline is 58.2%, which is higher than the average decline of 45.3% shown in Panel A. ADR = asymptotic dependence rate; Avg.Chi = average χ; Beta = CAPM beta; MES = marginal expected shortfall; Avg.Corr = average correlation across all other firms; Avg.Ken = average Kendall's Tau = across all other firms; Avg.Spr = average Spearman's Rho across all other firms; Assets = Total assets under management (see Appendix B Table B1); Lev = leverage (see Appendix B Table B1).

	N	Minimum	Maximum	Mean	Median	Standard deviation
Panel A						
ADR	27	0.038	0.923	0.658	0.769	0.255
Avg. Chi	27	0.013	0.429	0.302	0.349	0.127
Avg. Cor.	27	0.397	0.657	0.575	0.597	0.074
Beta	27	0.55	1.505	0.955	0.899	0.226
MES	27	1.289	3.355	2.094	1.904	0.524
Avg. Ken.	27	0.296	0.482	0.414	0.433	0.052
Avg. Spr.	27	0.423	0.656	0.571	0.596	0.065
Cumulative return	27	−0.996	0.306	−0.453	−0.425	0.344
Maximum return	27	−0.996	−0.187	−0.639	−0.604	0.207
Assets	27	14	2221	318	112	548
Lev.	27	2.747	9.204	6.319	6.231	1.549
Panel B						
ADR	27	0.192	1	0.801	0.885	0.188
Avg. Chi	27	0.072	0.465	0.366	0.401	0.1
Avg. Cor.	27	0.457	0.689	0.613	0.636	0.066
Beta	27	0.65	1.528	1.031	0.985	0.198
MES	27	1.469	3.444	2.384	2.29	0.477
Avg. Ken.	27	0.323	0.497	0.431	0.447	0.049
Avg. Spr.	27	0.458	0.671	0.592	0.611	0.06
Cumulative return	27	−0.988	0.502	−0.388	−0.419	0.33
Maximum return	27	−0.993	−0.096	−0.598	−0.604	0.204
Assets	27	14	2188	339	133	568
Lev.	27	2.776	26.646	8.739	8.389	4.634

Table 3
Regression of cumulative crisis period returns (July 2007–December 2008) on firm level indicators.
Notes: The dependent variable is the cumulative return from July 2007 to December 2008. The independent variables are: ADR = asymptotic dependence rate; Avg.Chi = average χ; Beta = CAPM beta; MES = marginal expected shortfall; Avg.Corr = average correlation across all other firms; Avg.Ken = Average Kendall's Tau, across all other firms; Avg.Spr = average Spearman's Rho across all other firms; Assets = Total assets under management (see Appendix B, Table B1); Lev = Leverage (see Appendix B, Table B1). Significance codes: $^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$. $N = 27$ since 2 depositories (Western Union and Sovereign Bancorp) drop from the sample for this period due to unavailable data. Estimation is performed via OLS.

	(1.1)	(1.2)	(1.3)	(1.4)	(1.5)	(1.6)	(1.7)	(1.8)	(1.9)
Int.	.626* (.308)	.618** (.296)	.341 (.417)	.413 (.418)	1.487*** (.457)	1.559*** (.462)	1.716*** (.499)	.504 (.379)	.471 (.368)
ADR	−.571** (.217)							−.629** (.243)	
Avg. Chi		−1.221*** (.430)							−1.362*** (.481)
Beta			−.158 (.311)					.175 (.307)	.207 (.302)
MES				−.104 (.140)					
Avg. Cor.					−2.350*** (.703)				
Avg. Ken.						−3.422*** (.988)			
Avg. Spr.							−2.760*** (.792)		
Assets	.003 (.123)	.007 (.121)	.002 (.153)	.028 (.159)	−.004 (.115)	−.009 (.114)	−.013 (.113)	−.029 (.137)	−.031 (.134)
Lev.	−.111** (.044)	−.112** (.043)	−.102** (.049)	−.104** (.049)	−.093** (.041)	−.094** (.040)	−.093** (.040)	−.111** (.044)	−.111** (.043)
R^2	.409	.431	.240	.250	.483	.495	.497	.417	.443
Adj. R^2	.332	.357	.141	.152	.415	.429	.431	.311	.342

	(1.10)	(1.11)	(1.12)	(1.13)	(1.14)	(1.15)	(1.16)	(1.17)
Int.	.546 (.382)	.510 (.372)	1.944** (.727)	1.933** (.861)	1.901*** (.676)	1.889** (.773)	2.139** (.770)	2.111** (.877)
ADR	−.611** (.248)		.453 (.557)		.335 (.480)		.348 (.479)	
Avg. Chi		−1.332** .4992		.816 (1.324)		.594 (1.106)		.605 (1.095)
Beta								
MES	.052 (.142)	.069 (.140)						
Avg. Cor.			−3.807* (1.926)	−3.679 (2.270)				
Avg. Ken.					−4.913** (2.359)	−4.766* (2.696)		
Avg. Spr.							−4.006** (1.892)	−3.858* (2.145)
Assets	−.024 (.146)	−.028 (.142)	−.014 (.117)	−.014 (.118)	−.020 (.116)	−.019 (.117)	−.026 (.116)	−.025 (.117)
Lev.	−.110** (.045)	−.110** (.044)	−.079* (.044)	−.081* (.045)	−.085* (.043)	−.086* (.043)	−.083* (.043)	−.084* (.044)
R^2	.412	.437	.498	.492	.506	.502	.509	.504
Adj. R^2	.305	.335	.407	.399	.416	.411	.419	.414

included in our study increased from June 2007 to December 2007. More importantly, the average decline in stock prices is larger during 2008.[15] Also, average peak-to-trough returns in both panels are of similar magnitude, which suggests that most of the sharp drops in stock prices occurred during 2008. Therefore, we conjecture that our measures will capture the variation in returns during the crisis better when the crisis is defined from January 2008 to December 2008.

Finally, we repeat the analysis defining stock return crisis performance by the peak-to-trough returns. By construction, peak-to-trough returns are more severe than the cumulative returns in both panels. Using peak-to-trough returns as the dependent variables, similarly to Billio et al. (2012), we aim to capture the most severe periods of crisis for each firm, and thus evaluate the predictive ability of tail dependence over the most severe shocks experienced by the firms in our sample during the crisis. Again, due to the focus of our firm-level measures on the most extreme losses, we conjecture that our measures will outperform other vulnerability indicators on the losses, we conjecture that our measures will outperform other vulnerability indicators on the prediction of this alternative measure of stock performance. We test this conjecture using the third set of regressions in this section.

When a financial institution is asymptotically dependent with many other institutions, and the average strength of this asymptotic dependence (the average χ of an institution) is larger, we expect the institution to be more vulnerable to financial crises. This is because strong tail dependence with other institutions leads to an increased likelihood of extreme losses when a significant number of those institutions are suffering tail losses, as is the case during financial crises.

In all regressions in Table 3, the dependent variable is the cumulative return of depository institutions during the July 2007 to December 2008 crisis period. We control for total assets under management and for the leverage of firms as of June 2007. Leverage is calculated as:

$$LVG = \frac{\text{book assets} - \text{book equity} + \text{market value of equity}}{\text{market value of equity}}$$

Book value of assets, book value of equity, and market value of equity were downloaded from Compustat.[16]

In regression (1.1), the explanatory variable *ADR* (*AsympDepRate*) is the percentage of other banks that are asymptotically dependent with the bank for which returns are being predicted. The coefficient of this variable is negative as expected, and it is statistically significant at the 5% significance level. The economic impact implied by this coefficient is quite large. Being asymptotically dependent with one more bank implies an additional stock price decline of 2.19%[17] during the crisis period. In regression (1.2), the explanatory variable *AvgChi* is the average χ of a bank taken across all other banks. The coefficient associated with this measure is also negative, and is statistically significant at the 1% significance level. A 1% increase in the average χ of a bank implies a 1.22%[18]

[15] One needs to adjust the statistics of cumulative return in Panel B for the shorter time period. The 18 month equivalent of 38.8% average decline is 58.2%, which is higher than the average decline of 45.3% shown in Panel A.

[16] Consistently with Acharya et al. (2010) and Billio et al. (2012), leverage is statistically significant in all regressions and has a negative impact on the crisis period returns, whereas total asset size does not have statistically significant effect on financial crisis returns.

[17] There are 27 banks in this regression since 2 firms drop due to unavailable data for this period. Therefore being asymptotically dependent with one other bank increases the AsympDepRate by 1/26, which results in a $-0.571/26 = -0.0219$ change in crisis return.

[18] A 1% increase in *AvgChi* results in a $0.01*(-1.221) = -0.0122$ change in crisis return.

Table 4
Regressions of cumulative crisis period returns (January 2008–December 2008) on firm level indicators.
Notes: The dependent variable is the cumulative return from January 2008 to December 2008. The independent variables are: ADR = asymptotic dependence rate; Avg.Chi = average χ; Beta, CAPM beta; MES = marginal expected shortfall; Avg.Corr = average correlation across all other firms; Avg.Ken = average Kendall's Tau across all other firms; Avg.Spr = average Spearman's Rho across all other firms; Assets = Total assets under management (see Appendix B, Table B1); Lev = leverage (see Appendix B, Table B1). Significance codes: $^{***}p<0.01$; $^{**}p<0.05$; $^{*}p<0.1$. N = 27 since 2 depositories (Western Union and Sovereign Bancorp) drop from the sample for this period due to unavailable data. Estimation is performed via OLS.

	(2.1)	(2.2)	(2.3)	(2.4)	(2.5)	(2.6)	(2.7)	(2.8)	(2.9)
Int.	.483** (.229)	.453** (.198)	.487 (.303)	.535* (.273)	.943* (.461)	.891* (.438)	.990* (.493)	.938*** (.331)	.828** (.304)
ADR	−.612** (.256)							−.595** (.244)	
Avg. Chi		−1.268** (.471)							−1.173** (.460)
Beta			−.494 (.283)					−.469* (.257)	−.409 (.256)
MES				−.248** (.115)					
Avg. Cor.					−1.596** (.754)				
Avg. Ken.						−2.142** (1.013)			
Avg. Spr.							−1.729** (.834)		
Assets	−.071 (.090)	−.063 (.087)	.005 (.104)	.015 (.100)	−.055 (.092)	−.056 (.092)	−.058 (.092)	.002 (.094)	.000 (.093)
Lev.	−.041*** (.011)	−.041*** (.011)	−.042*** (.012)	−.038*** (.011)	−.038*** (.011)	−.039*** (.011)	−.038*** (.011)	−.042*** (.010)	−.042*** (.010)
R^2	.510	.534	.459	.490	.487	.487	.484	.574	.582
Adj. R^2	.446	.473	.389	.424	.420	.420	.417	.497	.506

	(2.10)	(2.11)	(2.12)	(2.13)	(2.14)	(2.15)	(2.16)	(2.17)
Int.	.850*** (.296)	.762** (.273)	.717 (.499)	.384 (.585)	.720 (.458)	.438 (.524)	.745 (.527)	.406 (.611)
ADR	−.520** (.249)		−.453 (.396)		−.447 (.378)		−.460 (.378)	
Avg. Chi		−1.051** (.476)		−1.366 (.915)		−1.290 (.866)		−1.325 (.860)
Beta								
MES	−.202* (.110)	−.178 (.111)						
Avg. Cor.			−.607 (1.143)	.175 (1.395)				
Avg. Ken.					−.881 (1.465)	.054 (1.774)		
Avg. Spr.							−.666 (1.201)	.116 (1.446)
Assets	−.001 (.093)	−.003 (.093)	−.065 (.092)	−.065 (.090)	−.065 (.091)	−.064 (.090)	−.066 (.091)	−.064 (.090)
Lev.	−.039*** (.010)	−.039*** (.010)	−.040*** (.011)	−.041*** (.011)	−.040*** (.011)	−.041*** (.011)	−.040*** (.011)	−.041*** (.011)
R^2	.575	.583	.516	.534	.518	.534	.517	.534
Adj. R^2	.498	.507	.428	.450	.430	.449	.429	.450

additional decline in stock price during the crisis. The adjusted R^2's of the first two regressions are 40.9% and 43.1%, respectively.

In regression (1.3)–(1.7) we used several other measures from the finance and systemic risk literatures as explanatory variables. Regressions (1.3) and (1.4) use, respectively, the CAPM beta and the MES measure of Acharya et al. (2010) as explanatory variables.[19] As expected, high values on these measures seem to predict lower crisis returns. However, both are not statistically significant. In regressions (1.5)–(1.7) we analyze the ability of the average correlation measures (Pearson, Kendall and Spearman, respectively) proposed by Patro et al. (2013) to predict crisis returns. The three regressions produce negative and statistically significant coefficients for these correlation measures. Moreover, the adjusted R^2's of these regressions are higher than the adjusted R^2's of the regressions including the measures proposed in this study (regressions 1.1 and 1.2). Patro et al. (2013) demonstrated that these measures capture the downturns in the US financial system well. However, in that study, the authors did not perform this type of event study, in order to show the usefulness of their measures in identifying the firms that are more vulnerable to a financial crisis. Our study closes this gap and provides additional evidence in support of the measures developed by Patro et al. (2013).

In regressions (1.8)–(1.17) we include our average tail dependence measures together with other measures from the literature. As all these measures capture the systemic vulnerability of depository institutions in different ways, multicollinearity is likely to be a problem. We observe this in the results. When our measures are included together with the CAPM beta and the MES measure, the sign of the coefficients of the latter measures become positive, although not statistically significant. Similarly, when our measures are included together with average correlation measures proposed by Patro et al. (2013), the signs of the coefficients associated with our measures become positive, while, in most cases, the coefficients associated with the average correlation measures lose their statistical significance. We conclude that the counter-intuitive coefficient signs are the result of multicollinearity, but also that the average correlation measures capture the vulnerability to the financial crisis, defined as June 2007 to December 2008, somewhat better than our measures.

As mentioned in the introduction, the proposed measures in this study aim to provide complementary tools to address the limitations of the linear correlation coefficients in measuring dependence in the joint tails.[20] Particularly, we expect our measures to perform better in periods of severe stress. Statistically speaking, they should be better predictors of a firm's vulnerability in periods where more tail events were observed across the financial industry. To test this conjecture, we change the definition of the crisis period. Instead of the period used by Acharya et al. (2010) and Billio et al. (2012), July 2007 to December 2008, we calculate the average tail dependence measures as of the end of December 2007 and try to predict stock returns during the January 2008 to December 2008 period. For these regressions, the control variables asset size and the leverage are also calculated as of the end of December 2007. The results are presented in Table 4.

Similarly to the regressions previously discussed, higher values of our proposed measures, *ADR* and *AvgChi*, are predictive of lower stock returns. Their effects are statistically significant at the 1% level. Unlike in the prior regression, the negative effect of the MES measure on crisis returns is now also statistically significant at the

[19] Calculation of MES requires the choice of a "market" index. We used the CRSP value-weighted index as suggested by Acharya et al. (2010). The same index is used as the systematic factor in the calculation of CAPM beta.

[20] For a complete discussion of the limitations of correlation coefficients as measures of tail dependence, see Embrechts et al. (1999).

Table 5
Regressions of peak-to-trough crisis period return (January 2008–December 2008) on firm level indicators.
Notes: The dependent variable is the peak to trough return for the January 2008–December 2008 period. Independent variables are: ADR = asymptotic dependence rate; Avg.Chi = average χ; Beta = CAPM beta; MES = marginal expected shortfall; Avg.Corr = average correlation across all other firms; Avg.Ken = average Kendall's Tau across all other firms; Avg.Spr = average Spearman's Rho across all other firms; Assets = Total assets under management (see Appendix B, Table B1); Lev = leverage (see Appendix B, Table B1). Significance codes: $^{***}p < 0.01$; $^{*}p < 0.1$. $N = 27$ since 2 depositories (Western Union and Sovereign Bancorp) drop from the sample for this period due to unavailable data. Estimation is performed via OLS.

	(3.1)	(3.2)	(3.3)	(3.4)	(3.5)	(3.6)	(3.7)	(3.8)	(3.9)
Int.	.118 (.097)	.059 (.085)	−.145 (.175)	−.080 (.157)	.440* (.227)	.358 (.222)	.442* (.249)	.275* (.144)	.159 (.134)
ADR	−.560*** (.108)							−.553*** (.106)	
Avg. Chi		−1.073*** (.201)							−1.047*** (.203)
Beta			−.185 (.163)					−.161 (.111)	−.109 (.113)
MES				−.114* (.066)					
Avg. Cor.					−1.292*** (.371)				
Avg. Ken.						−1.640***(.513)			
Avg. Spr.							−1.338*** (.422)		
Assets	−.055 (.038)	−.049 (.037)	−.027 (.060)	−.016 (.057)	−.042 (.045)	−.044 (.046)	−.045 (.047)	−.030 (.041)	−.032 (.041)
Lev.	−.029*** (.005)	−.028*** (.005)	−.029*** (.007)	−.027*** (.006)	−.026*** (.006)	−.027*** (.006)	−.027*** (.006)	−.029*** (.005)	−.029*** (.005)
R^2	.772	.779	.532	.562	.676	.658	.656	.792	.788
Adj. R^2	.742	.750	.471	.505	.634	.613	.611	.754	.749

	(3.10)	(3.11)	(3.12)	(3.13)	(3.14)	(3.15)	(3.16)	(3.17)
Int.	.240* (.129)	.140 (.121)	.183 (.212)	−.081 (.248)	.153 (.195)	−.104 (.221)	.156 (.224)	−.138 (.257)
ADR	−.529*** (.108)		−.515*** (.168)		−.535*** (.161)		−.538*** (.161)	
Avg. Chi		−1.016*** (.211)		−1.270*** (.388)		−1.315*** (.365)		−1.316*** (.362)
Beta								
MES	−.067 (.048)	−.046 (.049)						
Avg. Cor.			−.169 (.485)	.354 (.591)				
Avg. Ken.					−.132 (.624)	.598 (.747)		
Avg. Spr.							−.096 (.511)	.494 (.608)
Assets	−.032 (.041)	−.033 (.041)	−.053 (.039)	−.051 (.038)	−.054 (.039)	−.051 (.038)	−.054 (.039)	−.051 (.038)
Lev.	−.028*** (.005)	−.028*** (.005)	−.028*** (.005)	−.029*** (.005)	−.028*** (.005)	−.029*** (.005)	−.028*** (.005)	−.029*** (.005)
R^2	.791	.787	.773	.782	.772	.785	.772	.785
Adj. R^2	.753	.749	.732	.743	.731	.746	.731	.746

1% level (regression 2.4). The three average correlation measures proposed by Patro et al. (2013) are also significant at the 1% level, and retain their negative coefficients (see regressions 2.5 through 2.7). The conjecture that our measures would likely perform better in a period reflecting a more severe crisis is confirmed. The regressions using our average tail dependence measures (regressions 2.1 and 2.2) have higher adjusted R^2 (44.6% and 47.3%, respectively) than the regressions using average correlation coefficients (42%). When our *ADR* measure is included in the regressions jointly with average correlation measures (regressions 2.12, 2.14 and 2.16), we observe that both variables have negative coefficients, but neither is statistically significant due to multicollinearity. When *AvgChi* is included together with the average correlation measures, we observe that the *AvgChi* has a negative, not statistically significant, coefficient, while the average correlation measures have positive coefficients. We conclude that tail dependence measures provide better explanatory power of stock return performance for the crisis period.

So far, we used the cumulative stock return during a unified stress period to measure the performance of firms during the crisis. Alternatively, the stress period can be defined separately for each firm as the period where the worst cumulative return was realized for that firm. One possible measure of performance that we adopt for this purpose is the minimum cumulative return during the financial crisis, which coincides with the *maxloss* variable used in Billio et al. (2012). This variable can also be interpreted as the peak-to-trough return. It is possible for a firm to perform well at the beginning of the exogenously defined crisis period and then suffer severe losses in market value later on, or vice versa. Thus, a period of gains may smooth out severe losses during the most stressful period for a firm. The alternative definition of the crisis period addresses this pitfall, by concentrating on the worse stretch of the financial crisis from each individual firm's perspective. Due to this definition of the crisis period, the crisis returns become even more severe and more dominated by tail events. Therefore, under this definition of crisis period, we conjecture that the average tail dependence measures proposed in this study can do a better job of identifying the firms that are more vulnerable to the crisis. We repeated the regressions discussed above for this definition of the crisis period. The results of these regressions are presented in Table 5. In these regressions, we calculated the independent variables as of the end of December 2007 and we used the peak-to-trough return during the January 2008 to December 2008 period as the dependent variable.

Both the *ADR* and the *AvgChi* measures perform well in these regressions, as they are statistically significant at the 1% level, and the regressions including these two variables (regressions 3.1 and 3.2) both have an adjusted R^2 above 74%. The average correlation measures also perform well, as they are statistically significant at 1%, and the regressions including them (regressions 3.5 through 3.7) have adjusted R^2 ranging from 61% to 63%. The difference in explanatory power reveals that the measures based on tail dependence do a better job in identifying the firms that are more vulnerable to the crisis. This finding can also be confirmed by the results of regressions including tail dependence measures together with average correlation measures (regressions 3.12 through 3.17). Again, multicollinearity is a problem and we observe some coefficients with unintuitive signs for the average correlation variables. However, in these regressions our proposed tail dependence measures perform better, and turn out to be statistically significant at the 1% level in all regressions. This contrasts with what we observed in Table 3, where the opposite happened. Overall, the results confirm our conjecture that tail dependence based measures perform better when the dependent variable is defined in a more severe way, and thus is more reflective of tail events.

Two key findings of this section should be emphasized. First, the correlation based measures developed by Patro et al. (2013) can be

Table 6
Descriptive statistics, when $\chi_{ij,t+1} > 0$.
Notes: $AD_{ij,t}$ = Binary indicator of asymptotic dependence between bank *i* and *j*. Balance-sheet variables are described in detail in Table B1 in Appendix B. $Similarity(Y_{i,t}, Y_{j,t}) = 1 - (\left|Y_{i,t} - Y_{j,t}\right|)/(Y_{i,t} + Y_{j,t})$.

Variables	*N*	Mean	St. Dev.	Min	Max
$\chi_{ij,t+1}$	10,880	.449	.100	.202	.731
$\chi_{ij,t}$	10,850	.406	.165	0	.731
$AD_{ij,t}$	10,850	.900	.301	0	1
Min{Ln(Assets)$_{i,t}$, Ln(Assets)$_{j,t}$}	10,600	10.91	.851	7.55	14.63
Similarity(Ln(Assets)$_{ij,t}$)	10,600	.943	.044	.776	1
Min{(Capital/Assets)$_{i,t}$,(Capital/Assets)$_{j,t}$}	10,600	.083	.018	.044	.144
Similarity((Capital/Assets)$_{ij,t}$)	10,600	.886	.097	.298	1
Min{(Cash/Assets)$_{i,t}$,(Cash/Assets)$_{j,t}$}	10,362	.046	.039	.003	.379
Similarity((Cash/Assets)$_{ij,t}$)	10,362	.598	.268	.017	1
Min{(Deposits/Liabilities)$_{i,t}$,(Deposits/Liabilities)$_{j,t}$}	9705	.618	.093	.334	.846
Similarity((Deposits/Liabilities)$_{ij,t}$)	9705	.904	.073	.535	1
Min{(Net Income/Assets)$_{i,t}$,(Net Income/Assets)$_{j,t}$}	10,585	.001	.005	−.047	.008
Similarity((Net Income/Assets)$_{ij,t}$)	10,585	.998	.004	.950	1
Min{(Non-Performing Assets/Assets)$_{i,t}$, (Non-Performing Assets/Assets)$_{j,t}$}	9480	.006	.006	0	.047
Similarity((Non-Performing Assets/Assets)$_{ij,t}$)	9480	.639	.280	0	1

used to identify firms that are more vulnerable to the occurrence of a systemic crisis. Second, the tail dependence based measures we proposed in this study can add further value beyond the information provided by correlation based measures and, thus, can be used as complementary tools in the analysis of systemic risk. Therefore, we recommend risk managers and bank regulators to monitor tail dependencies of firm stock returns in addition to correlations.

4. Predicting asymptotic dependence

In this section we analyze how balance sheet variables can be used to predict the strength of asymptotic dependence. Table 6 presents the descriptive statistics for the variables used in this section. As it is discussed in the previous sections, strong asymptotic dependence is usually associated with high systemic risk. Thus, we believe factors predictive of asymptotic dependence are factors to consider when defining systemically important institutions. In our analysis, we assume that the expected value of the strength of asymptotic dependence between two banks, $\chi_{ij,t+1}$, follows the following linear structure:

$$E[\chi_{ij,t+1}|\chi_{ij,t+1} > 0] = c_1 + c_2 \cdot 1\{\chi_{ij,t} = 0\} + \theta_{ij} + \tau_{t+1} + \rho\chi_{ij,t} + \beta X_{ij,t}$$

where θ_{ij} = bank pair fixed effect, τ_t = quarter fixed effect, and $X_{ij,t}$ = balance sheet variables for institutions *i* and *j* in quarter *t*.

We estimated this linear structure through a fixed effects regression. The aim of this regression is to identify predictors of the strength of asymptotic dependence, when asymptotic dependence exists, and thus we exclude observations for which $\chi_{ij,t+1}$ equals zero. We have adopted ordinary least squares (OLS) estimation, despite $\chi_{ij,t+1}$ being bounded between zero and one, because it allows us to include bank pair fixed effects – which prove crucial to properly identify the predictors of extremal dependence – while avoiding the incidental variables problem. This comes at the cost of having a slightly misspecified model (i.e. the error terms are not normally distributed, as they are assumed to be in OLS estimation).

The balance sheet variables we consider in our forecasting regressions are meant to differentiate financial institutions according to criteria typically considered in the literature. We distinguish institutions according to size, capital, liquidity, funding stability, earnings and asset quality using data from COMPUSTAT.[21] We measure size by the log of the bank's total assets; capital by equity capital divided by total assets; liquidity by cash divided by total assets; funding stability by the ratio between long-term deposits and liabilities; earnings by net income divided by total assets; and, finally, asset quality by the ratio between non-performing assets and total assets.

Assessing the effect of balance sheet variables on the measures of asymptotic dependence is challenging in our empirical setup because for each bank pair observation we have two sets of balance sheet variables. We have opted to include two measures for each variable, the minimum value for the indicator between the two institutions, and an index of similarity between the institutions. The similarity indexes are calculated by the expression[22]:

$$Similarity(Y_{i,t}, Y_{j,t}) = 1 - \frac{\left|Y_{i,t} - Y_{j,t}\right|}{Y_{i,t} + Y_{j,t}}$$

The two measures complement each other. The minimum measure allows us to assess how both institutions being of at least of a certain size, having at least a certain capitalization, and so on, affects the strength of asymptotic dependence. On the other hand, the similarity indexes allow us to analyze how much the differences between the institutions contribute to asymptotic dependence. In general, we expect increases in similarity indices to be associated with an increase in asymptotic dependence, as we expect similar institutions according to these indices to be more likely to be perceived as similar by the financial markets and, thus, more likely to experience tail co-movement.

It is likely that the factors described above are not the only factors that can predict asymptotic dependence. Also, the analysis presented in this section does not establish a causal relationship between these factors and the tail dependence of depository institutions. Nevertheless, we believe it is relevant to enquire whether these easily accessible balance sheet and income statement metrics can predict the development of extremal dependence.

We include time effects in the regression. Thus, the coefficients estimated capture how the variation of fundamentals across institutions within time periods correlates with asymptotic dependence. While this choice results in our regressions not being pure forecasting regressions, as part of the "time *t* + 1″ variation is absorbed through the time fixed effect, we believe this approach leads to more useful results. The usefulness of this analysis resides

[21] Data is used between 1995Q3 and 2011Q4. With a 6-year rolling window approach and stock price data starting from January 1990, the first Chi estimated corresponds to 1995Q4.

[22] Except in the case of Net Income over Assets, where similarity is equal to 1 – |(Net Income/Assets)$_{i,t}$ – (Net Income/Assets)$_{j,t}$|.

in the identification of factors predictive of institutions that are more likely to experience strong asymptotic dependence in future periods. Therefore, the analysis is enhanced by not confounding the effect of the factors in analysis with the effect of aggregate shocks on asymptotic dependence.

When calculating standard errors for our estimates, we have opted to use time-period clustering. We have experimented with different standard error specifications, including robust standard errors and clustering by pair of institutions, and concluded that time clustering produces the most conservative (largest) standard errors. The increase in standard errors associated with using time clustering likely results from different error term volatilities persisting for different time periods, even when time effects are accounted for. The following section discusses the results.

5. Results

Table 7 below presents the results of our regression.
These are the main findings of our analysis:

(a) Size, as measured by total assets, is a key predictor of asymptotic dependence. Pairs of institutions, where both institutions are large, experience stronger asymptotic dependence than pairs where at least one institution is small. This is an intuitive finding, and in line with larger financial institutions being more systemically important, as most systemic risk literature shows. Also, we find that similarity between the size of both institutions is predictive of higher extremal dependence.

(b) Capitalization is also a predictor of extremal dependence. Bank pairs where both banks have high capital ratios are less likely to show strong asymptotic dependence. This finding holds intuitive appeal to us. A possible reason is that well capitalized institutions are less likely to experience contagion resulting from severe losses of other well capitalized institutions. In regard to the similarity of capital levels, we find that this kind of similarity does not have a statistically significant effect on the level of asymptotic dependence between two institutions.

(c) Similar to capitalization, high liquidity is predictive of low extremal dependence. Bank pairs where both institutions are liquid, as measured by a large proportion of cash assets, are less likely to suffer from strong asymptotic dependence. Again, this result has intuitive appeal. The result suggests that banks with a good liquidity position are not likely to experience contagion from tail losses of other liquid banks. Also, similarity in the ratio of cash to assets is predictive of stronger extremal dependence.

(d) Higher stability of funding sources, as proxied by the share of deposits on total liabilities, also decreases asymptotic dependence. When both institutions enjoy a large share of deposit funding, they are less likely to suffer strong asymptotic dependence. Again, similarly to the effect of capitalization and liquidity on asymptotic dependence, this result is intuitive. Banks with stable funding sources are less likely to experience contagion due to tail losses of other banks with stable funding. In regard to the similarity of funding stability, we do not find a statistically significant effect of this kind of similarity on the asymptotic dependence between two banks.

(e) Profitability, as measured by the ratio between net income and total assets, does not significantly impact the strength of asymptotic dependence.

(f) Low asset quality is predictive of stronger extremal dependence. When both banks have high levels of non-performing assets, they experience stronger asymptotic dependence. Again, we believe this result is intuitive. When banks hold a high proportion of non-performing assets, it is likely that they are more vulnerable to contagion, and thus to experience losses simultaneously with other distressed banks. Similarity in regard to asset performance does not predict the strength of asymptotic dependence.

(g) Overall, higher similarity between institutions is predictive of stronger asymptotic dependence. We believe this result is intuitive, as similar banks according to these metrics are likely to be perceived similarly in the financial market, and thus more likely to experience large losses of market value simultaneously.

Table 7

$\chi_{ij,t+1}$ Forecasts for large US financial depositories.

Notes: $N = 9457$. $AD_{ij,t}$ = Binary indicator of asymptotic dependence between bank i and j. Balance-sheet variables are described in detail in Table B1 in Appendix B. $Similarity(Y_{i,t}, Y_{j,t}) = 1 - (\left|Y_{i,t} - Y_{j,t}\right|)/(Y_{i,t} + Y_{j,t})$. Time periods correspond to quarters. The regression follows a fixed-effects specification. Fixed-effects correspond to bank pairs. The regression also includes quarter dummies. Estimated standard errors, clustered by quarter, in parenthesis. $^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$.

Min{Ln(Assets)$_{i,t}$, Ln(Assets)$_{j,t}$}	.011** (.003)
Similarity(Ln(Assets)$_{ij,t}$)	.111*** (.035)
Min{(Capital/Assets)$_{i,t}$,(Capital/Assets)$_{j,t}$}	−.093* (.048)
Similarity((Capital/Assets)$_{ij,t}$)	−.001 (.007)
Min{(Cash/Assets)$_{i,t}$,(Cash/Assets)$_{j,t}$}	−.075*** (.026)
Similarity((Cash/Assets)$_{ij,t}$)	.011*** (.003)
Min{(Deposits/Liabilities)$_{i,t}$,(Deposits/Liabilities)$_{j,t}$}	−.033** (.015)
Similarity((Deposits/Liabilities)$_{ij,t}$)	.016 (.014)
Min{(Net Income/Assets)$_{i,t}$,(Net Income/Assets)$_{j,t}$}	.319 (.507)
Similarity((Net Income/Assets)$_{ij,t}$)	−.137 (.405)
Min{(Non-Performing Assets/Assets)$_{i,t}$,(Non-Performing Assets/Assets)$_{j,t}$}	1.698*** (.203)
Similarity((Non-Performing Assets/Assets)$_{ij,t}$)	.004 (.002)
$\chi_{ij,t}$	.481*** (.032)
$AD_{ij,t}$	−.186*** (.015)
Constant	.230 (.408)

6. Analysis of tail dependence for filtered returns

The upswings and downswings in the level of tail dependence between depository institutions can be the result of changes in the market systematic risk, as well as the result of changes in risk factors that are common to depository institutions, but not to the market as a whole, or even due to risk factors that are specific to a pair of banks. In this section, we analyze tail dependence of depository institutions, once systematic market factors are filtered out. For this purpose, we use the Fama and French (1993) three factor model, which includes an overall market factor, a firm size factor, and "book-to-market" factor. The first factor is the excess market return over the risk-free return. The second factor, SMB (small minus big), measures the excess returns of small capitalization stocks over big capitalization stocks. The third factor, HML (high minus low), measures the excess return of stocks with high book to market values over the stocks with low book to market values.[23] Similarly to our estimation of asymptotic dependence, we used rolling windows of six years to estimate the effects of the Fama-French factors on excess returns.

The evolution through time of the average R^2 of the Fama-French regressions across all firms is presented in Fig. 5. There is a clear trend of increasing explanatory power for the three factors, suggesting that the returns of depository institutions have become more tightly linked to the overall market movements over time. This finding is consistent with Patro et al. (2013) and De Nicolo and Kwast (2002). Also, this finding may partly explain the rise in asymptotic dependence and systemic risk in the banking system.

[23] We downloaded these three factors with daily frequency from Kenneth French's website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

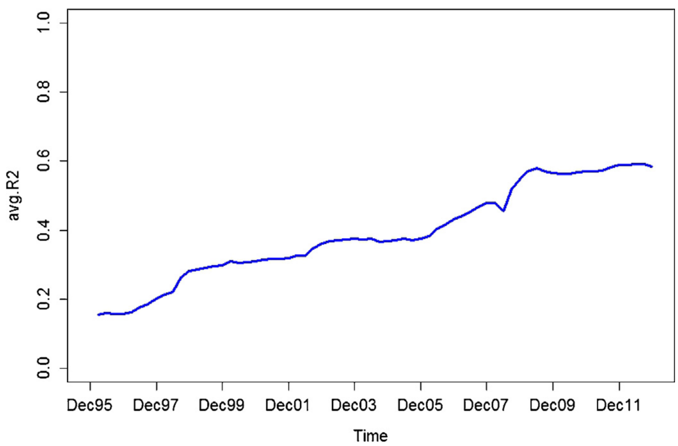

Fig. 5. Time series plot of average R^2 across all firms from the Fama-French regressions.

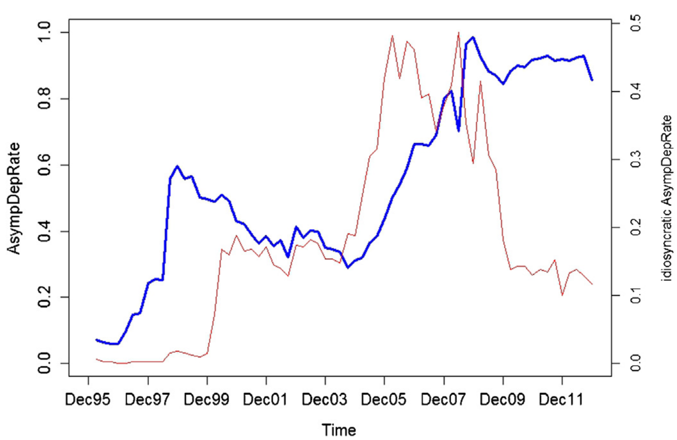

Fig. 6. Time series plot of *AsympDepRate* measure using raw returns (blue-left axis), and using the filtered returns (brown-right axis). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

However, it does not seem sufficient to fully explain the sharp rise in tail dependence during the crisis period.

Next, we calculate the systemic tail dependence measures proposed in Section 2, *AsympDepRate* and *AvgChi*, after the systematic component of the returns is filtered. Time series plots of these measures are presented in Figs. 6 and 7, respectively. In these plots, the measures calculated from raw returns are presented in blue and their value is displayed in the left axis, while measures calculated from filtered returns are presented in brown and their value is displayed in the right axis.

Since the systematic component is filtered out, the tail dependence of filtered returns is much lower in scale and more volatile. However after adjusting for scale, we observe a moderate amount of overlap between the two lines. Several observations are noteworthy.

First, the tail dependence of raw returns increased over the 1996–1998 period and spiked in 1998Q4, whereas the tail dependence of filtered returns stayed at low levels until 2000. This implies that the upswing in the tail dependence of depository institutions returns was mostly driven by market-wide systematic risk and not by factors specific to depository institutions. We believe that this makes sense because the market stress at this time did not originate within the banking sector. Instead, this time interval coincides with the Asian crisis, the Russian default and the failure of hedge fund Long Term Capital Management (LTCM).

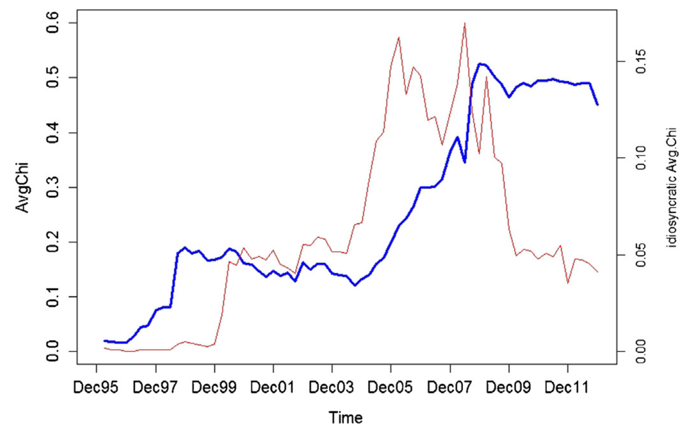

Fig. 7. Time series plot of the *AvgChi* measure using raw returns (blue-left axis) and using the filtered returns (brown-right axis). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Second, we observe that the tail dependence of the filtered returns increased sharply before the global financial crisis. The tail dependence of the raw returns followed the same pattern with some lag. This suggests that factors specific to the banks initially lead to the significant increase in tail dependence, and therefore to the increase in systemic risk, in the period leading up to the last financial crisis. We believe that this also makes sense because the recent financial crisis originated in the banking sector.

Third, we observe that the tail dependence of raw returns stay at very high levels even after the global financial crisis, even though the tail dependence of the filtered returns came down significantly in the second half of 2009. One possible explanation for this pattern is that while the factors initially driving tail dependence between depository institutions were specific to the banking system, after the second half of 2009 the factors became systematic in nature. We believe that further research is needed to fully understand the relationship between market-wide and industry-specific factors driving the tail dependence of stock returns.

7. Conclusion

We find that equity prices of large US depository institutions exhibit strong dependence even in their limiting joint extremes by calculating the extreme-value-theory-based tail dependence coefficient χ for each pair of institutions. Inspired by this finding, we propose two complementary systemic risk indicators. The first indicator measures the proportion of asymptotically dependent financial institution pairs (i.e., those with positive χ) to the total number of financial institution pairs in our sample. Our second indicator is the average of χ across all financial institutions. We also develop two complementary firm-level measures of average tail dependence. Our first firm-level measure is the proportion of institutions that are asymptotically dependent with that institution. Our second measure is the average χ of an institution across all the bank-pair combinations involving that institution.

Our analysis shows that the indicators of systemic risk tracked fairly well the periods of financial turmoil and stability. The ability of the systemic risk indicators we developed to predict future financial crises with substantial lead time may appear limited. Unfortunately, this is a feature shared by other indicators of systemic risk based on market return information. Nevertheless, our firm-level analysis shows that extremal dependence measures add value to the analysis of systemic risk, as they can contribute to the identification of institutions more vulnerable to financial crises.

We also studied whether our extremal dependence measures could contribute to the development of balance sheet indicators of vulnerability to financial crisis. To do so, we explored what institutional balance sheet information can be used to predict the strength of asymptotic dependence. We consider criteria identified in the systemic risk literature and designated by the Dodd Frank Act (section 113) in the United States and the Basel Committee internationally. Size, capitalization, liquidity, funding sources and asset quality of financial institutions are good predictors of asymptotic dependence. Also, high similarity between institutions is predictive of high asymptotic dependence.

Lastly, we calculated systemic risk measures from filtered returns obtained by applying the three-factor model of Fama-French. These adjusted measures revealed interesting findings. For example, while the increase in systemic risk around 1998 was almost entirely driven by systematic market factors, the increase during the recent financial crisis was driven in a large part by factors not captured by the Fama-French model. We believe this is an intuitive finding since the crisis in 1998 did not originate in the US banking sector, while the recent financial crisis was triggered by the subprime lending of US banks. Also, we find that after the second half of 2009 the factors driving the tail dependence among banks became systematic in nature.

In this paper, we focused on large US depository institutions, leaving to future research possible extensions to other types of financial institutions and geographic areas. We believe the proposed measures of extremal dependence have the potential to inform the prudential supervision of systemically important firms, an area currently of great relevance in supervisory policy.

Appendix A. Power law and its estimation with the Hill's estimator

Gnedenko (1943) derived that all sub-exponential distributions follow a power law beyond a high threshold, which can be written as:

$$\bar{F_T}(t) = Pr(T > t) \cong l(t)t^{-1/\xi} \text{ for } t > u \tag{A.1}$$

where l(t) is a slowly varying function, u is a high threshold and ξ is the tail parameter. The tail parameter can be estimated by the Hill's estimator. The estimation of the tail parameter relies on the slowly varying function l(t) being approximately constant above a high threshold u. Due to this approximation, we have:

$$Pr(T > t|T > u) = \frac{l(t)t^{-1/\xi}}{l(u)u^{-1/\xi}} = \left(\frac{t}{u}\right)^{-1/\xi} \tag{A.2}$$

The log-likelihood function of the N_u observations above the threshold u can be written as:

$$LogL(\xi, T) = \sum_{j=1}^{j=N_u} \left(-\left(\log\xi + \log u\right) - \left(1 + \frac{1}{\xi}\right) \log\left(\frac{t_j}{u}\right)\right) \tag{A.3}$$

The Hill's estimator is the closed form solution to the maximization of this likelihood. The Hill's estimator of ξ and the variance of this estimator are, respectively:

$$\hat{\xi}_{mle} = \frac{1}{N_u}\sum_{j=1}^{j=N_u} \log\left(\frac{t_j}{u}\right) \tag{A.4}$$

$$\hat{\sigma}_{\xi}^2 = \frac{\hat{\xi}^2}{N_u} \tag{A.5}$$

Inference regarding the tail parameter can be performed due to the asymptotic properties of the Hill's estimator, as shown in Hill (1975).

Substituting $t = u$ in (A.1), and estimating Pr($T > u$) non-parametrically with N_u/N, an estimator for the constant approximating the value of the slowly varying function above u can be obtained as:

$$l(t) = \frac{N_u}{N} u^{1/\xi} \text{ for } t > u \tag{A.6}$$

And thus, when $\xi = 1$, the estimate of χ is equal to:

$$\hat{\chi} = \frac{N_u}{N} u$$

In a simulation study, McNeil and Frey (2000) found that above the 90th quantile of fat tail distributions, the bias in tail estimation becomes reasonable. In our analysis, we chose the 95th quantile of T as the high threshold u, so that the bias is even smaller.[24]

Appendix B. Additional data details

As Compustat and CRSP each account for mergers and acquisitions differently, share prices were corrected using a stacking method. For example, when Society Corporation acquired KeyCorp (referred to here as "Old KeyCorp") on March 2, 1994, the merged entity adopted KeyCorp's name and continued to trade with Old KeyCorp's ticker. CRSP and Compustat maintain unique identifiers for commercial entities; the CRSP identifier (PERMNO) relates to the trading equity, while the Compustat identifier (GVKEY) applies to the organization. The CRSP identifier for KeyCorp did not change following acquisition, even though Old KeyCorp had been acquired by Society Corporation. For this research, the primary identifier is Compustat's GVKEY, and in case of acquisitions, the acquiring entity's earlier stock prices are stacked on the post-merger share prices. In the case of KeyCorp, Society Corporation's closing share prices before March 2, 1994 were stacked on KeyCorp's closing share prices after March 2. Share prices are further adjusted for stock splits by dividing the price with a cumulative factor provided by CRSP.

Table B1

Regression variable(s)	Compustat variable(s) & derivation	Description
Assets, Ln(Assets)	ATQ	End-of-quarter assets in millions of dollars
Leverage	(ATQ-SEQQ + MKVALTQ)/ MKVALTQ	Total assets (ATQ) less parent's stockholder equity (SEQQ) plus market capitalization (MKVALTQ), as a share of market capitalization value
Capital/Assets	TEQQ/ATQ	Total equity as a share of total assets; total equity is the sum of common/ordinary equity, preferred stock, and nonredeemable, noncontrolling interest in subsidiaries
Cash/Assets	CHEQ/ATQ	Cash and all securities readily transferable to cash as a share of total assets

[24] Threshold selection is a bias-variance trade-off. Selection of a very low threshold may cause the asymptotic results for extremes not to hold and, therefore, may lead to biased parameters. On the other hand, the choice of a very high threshold will cause the parameter estimates to have a big variance, since there are only few observations above the threshold.

Table B1 (*Continued*)

Regression variable(s)	Compustat variable(s) & derivation	Description
Deposits/Liabilities	DPTCQ/LTQ	Total demand, savings, and time deposits held on account for individuals, partnerships, and corporations as a share of total liabilities
Net Income/Assets	NIQ/ATQ	Income or loss reported by a company after expenses and losses have been subtracted from all revenues and gains for the fiscal period, including extraordinary items and discontinued operations, as a share of total assets
Nonperforming Assets/Assets	NPATQ/ATQ	Loans and leases carried on a non-accrual basis, 90 days past due (both accruing and nonaccruing), renegotiated loans, real estate acquired through foreclosure, and repossessed movable property as a share of total assets

Table B1 presents the definitions and sources of the quarterly financial variables used in Tables 6 and 7.

Table B2 provides an an account of the institutions that do not have returns for the entire sample period.

Table B2

Company	Obs.	Description
Commerce Bancorp	4597	Commerce Bancorp was acquired by TD Bank in 2008.
Hudson City Bancorp	3389	Hudson City Bancorp first issued stock in 1999. Our sample begins in 1990
Marshall & Ilsley	5421	Marshall & Ilsley was acquired by Bank of Montreal in 2011
National City Corp	4790	National City Corp was acquired by PNC in 2009
New York Community Bank	4810	New York Community Bank (formerly Queen County Savings Bank) did not have publicly traded stock before 1993
Sovereign Bank	4807	Banco Santander acquired Sovereign Bank in 2009. Sovereign Bank was delisted after the acquisition
Unionbancal Corp.	4750	The Bank of Tokyo-Mitsubishi acquired all outstanding shares of Unionbancal Corporation in November 2008
Wachovia	4790	Wachovia merged with Wells Fargo in 2008
Washington Mutual	4724	Washington Mutual failed in 2008
Western Union	1572	Western Union's IPO was in 2006

References

Acharya, V., Pedersen, L., Philippon, T., Richardson, M., 2010. Measuring Systemic Risk. Federal Reserve Bank of Cleveland, pp. 10–12.

Adrian, T., Brunnermeier, M.K., NBER Working Papers 17454 2011. CoVaR.

Basel Committee, 2011. Globally systemically important banks: Assessment methodology and the additional loss absorbency requirement. http://www.bis.org/publ/bcbs255.pdf

Brownless, C., Engle, R., 2012. Volatility, correlation, and tail for systemic risk measurement. http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1611229

Billio, M., Getmansky, M., Lo, A.W., Pelizzon, L., 2012. Econometric measures of connectedness and systemic risk in the finance and insurance sectors. J. Financ. Econ. 104, 535–559.

Chan-Lau, J.A., Mathieson, D.J., Yao, J.Y., 2004. Extreme contagion in equity markets. IMF Staff Pap. 51 (2), 386–408.

De Nicolo, G., Kwast, M.L., 2002. Systemic risk and financial consolidation: are they related? J. Bank. Finance 26 (5), 861–880.

DiTraglia, F., Gerlach, J., 2013. Portfolio selection: an extreme value approach. J. Bank. Finance 37 (2), 305–323.

Embrechts, P., McNeil, A., Straumann, D., 1999. Correlation: pitfalls and alternatives. Risk Mag., 69–71.

Ergen, I., 2014. Tail dependence and diversification benefits in emerging market stocks: an extreme value theory approach. Appl. Econ. 46 (19), 2215–2227.

Fama, E., 1965. The behavior of stock market prices. J. Bus. 38, 34–105.

Fama, E., French, K., 1993. Common risk factors in the returns on stocks and bonds. J. Financ. Econ. 33 (1), 3–56.

Gnedenko, B., 1943. Sur la distribution limite du terme maximum d'une serie aleatoire. Ann. Math. 44, 423–453.

Hartmann, P., Straetmans, S., De Vries, C.G., 2004. Asset market linkages in crisis periods. Rev. Econ. Stat. 86 (1), 313–326.

Hill, B.M., 1975. A simple general approach to inference about the tail of a distribution. Ann. Stat. 3 (5), 1163–1174.

Huang, X., Zhou, H., Zhu, H., 2009. A framework for assessing the systemic risk of major financial institutions. J. Bank. Finance 33 (11), 2036–2049.

Ledford, A.W., Tawn, J.A., 1996. Statistics for near independence in multivariate extreme values. Biometrika 83 (1), 169–187.

Ledford, A.W., Tawn, J.A., 1997. Modeling dependence within joint tail regions. J. R. Stat. Soc. Ser. B 59 (2), 475–499.

Ledford, A.W., Tawn, J.A., 1998. Concomitant tail behavior for extremes. Adv. Appl. Probab. 30 (1), 197–215.

Longin, F., Solnik, B., 2001. Extreme correlation of international equity markets. J. Finance 56 (2), 649–676.

Mandelbrot, B., 1963. The variation of certain speculative prices. J. Bus. 36, 394–419.

McNeil, A.J., Frey, R., 2000. Estimation of tail related risk measures for heteroscedastic financial time series. J. Empir. Finance 7, 271–300.

Moore, K., Zhou, C., DNB Working Paper 347 2012. Identifying systemically important financial institutions: size and other determinants.

Nelsen, R.B., 2006. An Introduction to Copulas, 2nd ed. Springer, New York.

Patro, D.K., Qi, M., Sun, X., 2013. A simple indicator of systemic risk. J. Financ. Stab. 9 (1), 105–116.

Poon, S., Rockinger, M., Tawn, J., 2003. Modelling extreme-value dependence in international stock markets. Stat. Sin. 13, 929–953.

Poon, S., Rockinger, M., Tawn, J., 2004. Extreme value dependence in financial markets: diagnostics, models, and financial Implications. Rev. Financ. Stud. 17 (2), 581–610.

Sibuya, M., 1960. Bivariate extreme statistics I. Ann. Inst. Stat. Math. 11, 195–210.

Starica, C., 1999. Multivariate extremes for models with constant conditional correlations. J. Empir. Finance 6 (5), 515–553.

Trivedi, P.K., Zimmer, D.M., 2007. Copula modeling: an introduction for practitioners. Found. Trends Econ. 1 (1), 1–111.

Zhou, C., 2010. Are banks too big to fail? Measuring systemic importance of financial institutions. Int. J. Central Bank. 6 (4), 205–250.